Solving Systems of Linear Equations by Substitution

Focus on ...

- solving systems of linear equations algebraically using substitution

This year, the environmental club is having a fundraiser. Members are selling compact fluorescent light bulbs and 100% organic T-shirts with natural dyes. The price of one T-shirt is three times the price of one light bulb. You purchase two shirts and one light bulb for $42. If you lost your receipt, how could you determine the unit price for a light bulb and for a T-shirt?

Investigate Solving Systems of Linear Equations by Substitution

In the following balance diagrams, each block is identical in mass. Each cone is identical in mass.
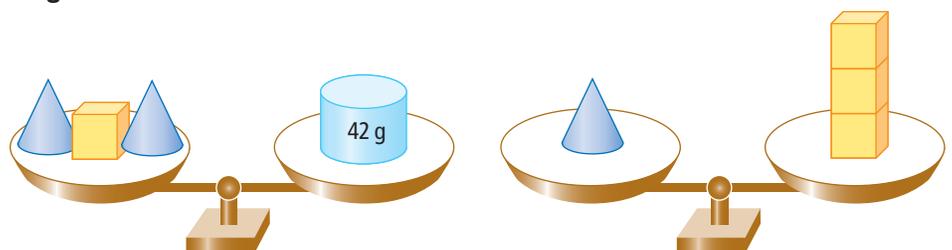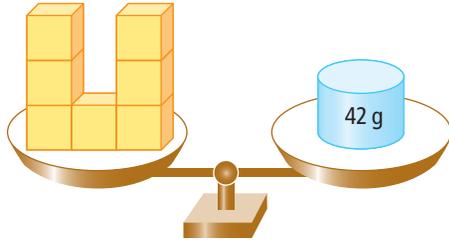
Diagram 1

Diagram 2

1. a) Describe how Diagram 2 relates to Diagram 1.

2. Describe how you could determine the mass of one block from Diagram 2. What is the mass of one block?

3. What is the mass of one cone? How did you determine your answer?

4. Write an equation for each balance scale in Diagram 1. Remember to state what your variables represent.

5. Write an equation for Diagram 2.

6. Suppose the mass of a block represents the cost of one light bulb, the mass of a cone represents the cost of one T-shirt, and each gram represents one dollar. Use algebra to show how you can determine the cost of one light bulb and the cost of one T-shirt for the scenario on page 468.

7. Reflect and Respond Use diagrams to explain how to determine the mass of a single pyramid and the mass of a single cylinder for the following scenario.
  • Five pyramids and three cylinders have a mass of 44 g.
  • Two pyramids have the same mass as one cylinder.

pyramid   cylinder

8. Use algebra to determine the mass of one pyramid, p, and the mass of one cylinder, c.

9. Describe a situation where using a diagram is less effective than using algebra.

Link the Ideas

The skill of substituting algebraic expressions is used regularly in math and science. The **substitution method** can provide a quick solution to a linear system.

Solve the following linear system.
$$4x + 5y = 26$$
$$3x = y - 9$$

First, solve for y in $3x = y - 9$.
$$3x + 9 = y - 9 + 9$$
$$3x + 9 = y$$

Substitute $3x + 9$ for y in $4x + 5y = 26$.
$$4x + 5(3x + 9) = 26$$
$$4x + 15x + 45 = 26$$
$$19x + 45 = 26$$
$$19x + 45 - 45 = 26 - 45$$
$$19x = -19$$
$$\frac{19x}{19} = \frac{-19}{19}$$
$$x = -1$$

Substitute -1 for x in $3x = y - 9$.
$$3(-1) = y - 9$$
$$-3 = y - 9$$
$$-3 + 9 = y - 9 + 9$$
$$6 = y$$

Example 1   Solve a System of Linear Equations by Substitution

Admission to the 2009 Abbotsford International Airshow cost $80 for a car with two adults and three children. Admission for a car with two adults cost $50. Use algebra to determine the cost for one child and the cost for one adult. There was no charge for the vehicle or parking.

Solution

Let C represent the cost for one child, in dollars.
Let A represent the cost for one adult, in dollars.

For the first car, $2A + 3C = 80$.
For the second car, $2A = 50$.

Determine the admission prices.

Method 1: Solve for *A* First

The second equation has only one variable. So, determine the cost for one adult first.

$$2A = 50$$
$$A = 25$$

Solve for C by replacing A with 25.

$$2A + 3C = 80$$

You can also replace 2*A* with 50.

$$2(25) + 3C = 80$$
$$50 + 3C = 80$$
$$3C = 30$$

If $50 + 3C = 80$, how do you know that $3C = 30$?

$$C = 10$$

What does the value 10 represent?

Method 2: Solve for *C* First

Use substitution.

$$50 + 3C = 80$$

Replace 2*A* with 50 in the equation.

Solve for C.

$$50 + 3C = 80$$
$$3C = 30$$
$$C = 10$$

Solve for A by replacing C with 10.

$$2A + 3(10) = 80$$
$$2A + 30 = 80$$
$$2A = 50$$

If $2A = 50$, how do you know that $A = 25$?

$$A = 25$$

What does the value 25 represent?

Check:
Substitute into the original equations, $2A + 3C = 80$ and $2A = 50$.

Left Side	Right Side
$2A + 3C$	80
$= 2(25) + 3(10)$	
$= 50 + 30$	
$= 80$	

You can also check your solution by graphing.

Left Side = Right Side

Left Side	Right Side
$2A$	50
$= 2(25)$	
$= 50$	

Left Side = Right Side

The admission price is $10 for a child and $25 for an adult.

Your Turn

Solve the following linear system algebraically using substitution.

$$3x + 5y = 27$$
$$4x = 16$$

Example 2   Isolate a Variable Before Solving by Substitution

At a dance recital, there were 220 people. Tickets cost $9 for an adult and $6 for a child. The dance school collected $1614 in ticket sales. How many adults and how many children attended the recital?

Solution

Let a be the number of adults at the recital.
Let c be the number of children at the recital.
Write an equation that represents the total number of adults and children.

$a + c = 220$ ①   **You can number the equations to make it easier to refer to them throughout the solution.**

Write an equation that represents the amount collected by the dance school.

$9a + 6c = 1614$ ②

Isolate a variable in one of the equations.

Method 1: Isolate the Variable c in ①

$a + c = 220$
$\quad c = 220 - a$ **You can isolate the variable c in ① easily because the coefficient of the variable is 1.**

Substitute for c in ②.

$9a + 6(220 - a) = 1614$
$9a + 1320 - 6a = 1614$
$\quad\quad 3a + 1320 = 1614$
$\quad\quad\quad\quad\quad 3a = 294$
$\quad\quad\quad\quad\quad a = 98$ **What does the value 98 represent?**

Substitute the number of adults into ① to finish solving the system.

$98 + c = 220$
$\quad\quad c = 122$ **What does the value 122 represent?**

Method 2: Isolate the Variable *c* in ②

$$9a + 6c = 1614$$
$$6c = 1614 - 9a$$
$$c = 269 - \frac{9}{6}a$$

Compare isolating c in Method 2 with isolating c in Method 1. Why does Method 2 take more steps?

Substitute for c in ①.

$$a + \left(269 - \frac{9}{6}a\right) = 220$$
$$-\frac{3}{6}a + 269 = 220$$
$$-\frac{1}{2}a + 269 = 220$$
$$-\frac{1}{2}a = -49$$
$$-\frac{1}{2}a(-2) = -49(-2)$$
$$a = 98$$

Substitute a in ① to finish solving the system.

$$98 + c = 220$$
$$c = 122$$

Check:
Substitute into ① and ②.

Left Side	Right Side
$a + c$	220
$= 98 + 122$	
$= 220$	

Left Side = Right Side

Left Side	Right Side
$9a + 6c$	1614
$= 9(98) + 6(122)$	
$= 882 + 732$	
$= 1614$	

Left Side = Right Side

At the dance recital, there were 98 adults and 122 children in attendance.

Which method do you prefer? Why? How might the solution be different if you isolated the variable a instead of c?

Your Turn

Solve the following linear system algebraically using substitution. Check your solution.

$$2x + y = 13$$
$$x - 0.4y = -16$$

Key Ideas

- You can solve systems of linear equations algebraically using substitution.
  - Isolate a single variable in one of the two equations.
  - Where possible, choose a variable with a coefficient of 1.

    Solve the linear system.

    $3x + 2y = -11$ ①
    $-2x + y = 12$ ②

    Isolate the variable y in ② since its coefficient is 1.

    $y = 12 + 2x$

    Substitute the expression for y in ①.

    $3x + 2(12 + 2x) = -11$
    $3x + 24 + 4x = -11$
    $7x + 24 = -11$
    $7x = -35$
    $x = -5$

  - Substitute the solution for the first variable into one of the original equations. Solve for the remaining variable.

    $-2(-5) + y = 12$
    $10 + y = 12$
    $y = 2$

  - Check your answer by substituting into both original equations.

Check Your Understanding

Practise

1. Solve the following systems of linear equations by first substituting for y.

a) $y = 3x + 2$
$x + y = 14$

b) $y = -3x$
$y - x = 24$

c) $y = x - 7$
$x + y = 17$

2. Solve the following linear systems by substitution.

a) $2x - 3y = 10$
$x + y = 0$

b) $m = 8j$
$-m + 2 = -7j$

c) $2k = 6n + 9$
$n - 2k = -4$

3. Solve each linear system two ways. First, solve by isolating x. Then, solve by isolating y. For each linear system, explain which method you prefer and why.

a) $y = 0.3x - 5$
$1.7x + y = 9$

b) $y = 10 - 2.2x$
$5x + y = 70$

c) $\frac{x}{2} = 5 - y$
$x + y = 7$

4. Solve the following systems of linear equations. Check your answers.

a) $y = \dfrac{1}{3}x - 5$

$x - \dfrac{y}{5} = 13$

b) $\dfrac{y - x}{2} = 5$

$x + \dfrac{3}{4}y = 4$

c) $3y = \dfrac{1}{3} - \dfrac{2x}{3}$

$x + \dfrac{3y}{2} = 12$

5. Jaret and Helen are going to solve the following system of equations.

$3y = 2x - 6$
$5x + 2y = 29$

Jaret decides to first isolate x in the first equation.

It has a coefficient of 2, and it is easy to divide by 2.

Helen decides to first isolate y in the first equation.

It is almost isolated already.

a) Solve this system using Jaret's method. Then, solve it using Helen's method.

b) Explain which method you prefer.

6. The sum of two numbers is 20. Twice one number is four more than four times the other. Write a system of linear equations and determine both numbers.

7. The graph represents the solution to the following linear system.

$y = 2x + 3$
$6x + 3y = 13$

a) What are the coordinates of the point of intersection?

b) Solve the linear system using the substitution method.

c) Compare your answers for parts a) and b). What is the advantage of the algebraic approach?
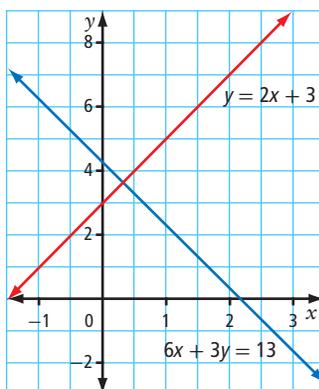
8. a) Solve the following system of linear equations by substitution.

$0.1y = 0.3x - 1.5$

$x - 0.2y = 5.6$

b) Multiply both sides of each equation by 10 first. Then, solve by substitution. How does the multiplication by 10 help you to solve?

c) Solve the system graphically.

d) Which of the three methods do you prefer? Explain.

Apply

Solve problems 9 to 20 using the substitution method. Check your answers.

9. An 82-m cable is cut into two pieces. One piece is 18 m longer than the other. What is the length of each piece?

10. Whitehorse, YT, has three times as much snowfall each year as Vancouver, BC. The total combined snowfall for these two cities is approximately 192 cm. What is the snowfall in each city?

11. Alaina has $72 and earns $6 each day. Joel has $48 and earns $8 each day. In how many days will Joel have as much money as Alaina?

12. In Manitoba, teenagers watch approximately 11 fewer hours of TV each week than adults do. The sum of the hours watched per week for an adult and a teenager is about 37 h. Approximately how many hours per week do teenagers watch?

13. A young Colorado blue spruce tree is growing at a rate of 20 cm per year. Currently, it is 244 cm tall. A 300-cm tall white spruce tree is growing at a rate of 12 cm per year. In how many years will the two trees be the same height?

244 cm

300 cm

14. Rory's grandmother is 58 years older than Rory. In 5 years, they plan to have a party to celebrate that their ages have a sum of 100. How old are they now?

15. (**Unit Project**) A section of a local habitat was damaged during a storm. A local company wishes to preserve the wetland and ensure water quality. The organizers decide to replace some of the bushes and trees. They place two orders with a nursery.

• One order is for 40 bushes and 12 trees. It totals $1484.
• The other order is for 25 bushes and 18 trees. It totals $1421.
Create and solve a system of linear equations to determine the cost of one bush and the cost of one tree.

16. In Amir's coin collection, the number of dimes is one more than three times the number of nickels. The total number of nickels and dimes is 69.

a) Amir also has 40 quarters in his collection. How many more dimes than quarters does he have? How many more quarters than nickels does he have?

b) Explain why you can solve this problem without knowing the value of a nickel, dime, or quarter.

17. Students from two schools went on a trip to the Wascana Waterfowl Park in Regina, SK. They learned about the value of conserving natural resources. School A rented and filled 8 vans and 8 buses with 400 people. School B rented and filled 4 vans and 1 bus with 68 people. Every van had the same number of students in it, as did every bus. Determine the number of students in each van and in each bus.

18. Water is used during each step of bread manufacturing, from processing the wheat into flour to making the bread itself. Less water is used to produce a slice of whole-wheat bread than a slice of white bread.

- To produce 60 slices of whole-wheat bread and 10 slices of white bread, 2080 L of water are used.
- To produce 20 slices of whole-wheat bread and 50 slices of white bread, 2560 L of water are used.

How many litres of water are used to produce one slice of whole-wheat bread? How many litres of water are used to produce one slice of white bread?

19. Andrew has a collection of 132 coins that consists of quarters and loonies. The value of the collection is $77.25. He wants to determine the number of quarters and the number of loonies he has without counting.

$q + n = 132$ and $0.25q + 1.00n = 77.25$, where q is the number of quarters and n is the number of loonies.

$0.25q + 1.00(132 - q) = 77.25$	Step 1
$100(0.25q) + (1.00)(132 - q) = (100)77.25$	Step 2
$25q + 1.00(132 - q) = 7725$	Step 3
$25q + 132 - q = 7725$	Step 4
$24q = 7593$	Step 5
$q = \dfrac{7593}{24}$	Step 6

Andrew stopped when he realized he had made a mistake.

a) How did Andrew know $q = \dfrac{7593}{24}$ could not be correct?

b) Identify where Andrew made the first error in his work.

c) Solve the linear system correctly.

20. Solve the following linear systems. Express your answers as fractions.

a) $y = \dfrac{1}{7}x - 2$
$5x = 3y + 1$

b) $\dfrac{1}{3}x + 4y = \dfrac{47}{15}$
$5x + 2y = 3.5$

Extend

21. On a graph, a line with the equation $y = mx + b$ passes through the points (2, 7) and (5, 1). Solve a linear system algebraically to determine the values of m and b. Include a diagram of the coordinate plane with your solution.

22. Arman walks to the train station at 5 km/h. He misses his train by 1 min. If he had run at 10 km/h, he would have had 2 min to spare. How far is it to the station?

23. In science, students frequently study simple electrical circuits. The relationship between the resistance of a circuit, R, the current, I, and the voltage, V, is $V = RI$. The relationship between the power, P, the voltage, and the current is $P = VI$. Use substitution to write a formula that determines the power from the resistance and the current. Show your work.
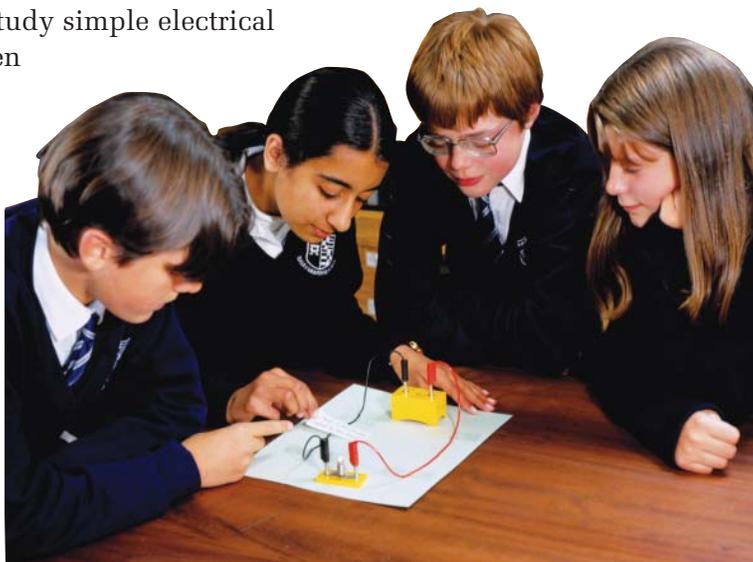
24. Use substitution to show that the linear system $y = 2x + 5$ and $2y - 4x = -15$ has no solution. How do you know there is no solution?

Create Connections

25. Compare solving a linear system by substitution to solving graphically.
  a) How are the methods similar?
  b) How are the methods different?

26. Choose a question from section 9.1 that has a system of linear equations.
  a) Solve the system using a graphical approach.
  b) Compare the graphical solution to the solution using substitution. Which method do you prefer for solving this system of equations? Explain.